Collective behaviour

Parameter inference for model comparison

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What's it all about?

Where's the maths?

- Many agent-based models (ABMs) have been proposed to try explain collective motion
- ABMs follow a Lagrangian approach, where behaviour is modelled at an individual level
- Simple rules compound to create complex behaviour

Where do we fit in?

- Loads of different ABMs have been proposed
- Very little quantitative comparison between model and data
- Working in a Bayesian paradigm we to seek carry out rigorous model verification / falsification

Our dataset

- Courtesy of Hayley Moore of the CDT programme
- Tracks positions of flocking sheep through time
- Raw data
- Extracted data

The model

Positional update:

$$\boldsymbol{x}_{i,t+1} = \boldsymbol{x}_{i,t} + \boldsymbol{v}_{i,t}$$

Directional update:

$$\theta_{i,t+1} = \text{atan2}\left(\sum_{j=1}^{N} \omega_{ij,t} \sin\theta_{j,t}, \sum_{j=1}^{N} \omega_{ij,t} \cos\theta_{j,t}\right) + \epsilon_{i,t}$$

where $\epsilon_{i,t} \sim N(0, \sigma_{Y_i})$ and

$$\omega_{ij,t} = \frac{1}{\sqrt{2\pi\sigma_{X_i}{}^2}} \exp\left(\frac{-d_{ij,t}^2}{2\sigma_{X_i}{}^2}\right)$$

In summary

The model:

- Each individual's behaviour is controlled by two parameters, σ_{X_i} and σ_{Y_i}
    - σ_{X_i} controls how strongly agent *i* interacts with neighbours
    - σ_{Y_i} controls how much noise agent *i* experiences

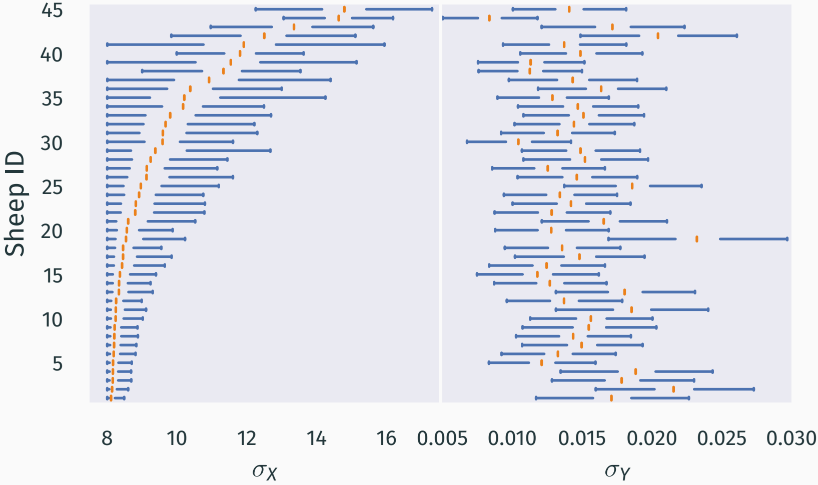
Our goal:

- Infer values of σ_{X_i} and σ_{Y_i} for every sheep in our dataset

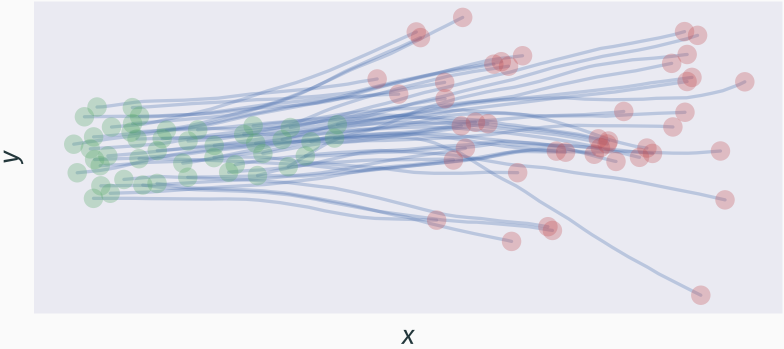
A black box solution with STAN

- Stan is a probabilistic programming language, similar to BUGS and JAGS.
- Implements NUTS algorithm — a variant of HMC
    - Input: data and model specification
    - Output: posterior densities of parameters

Results: posterior densities



Results: forward simulations



Current model fitting

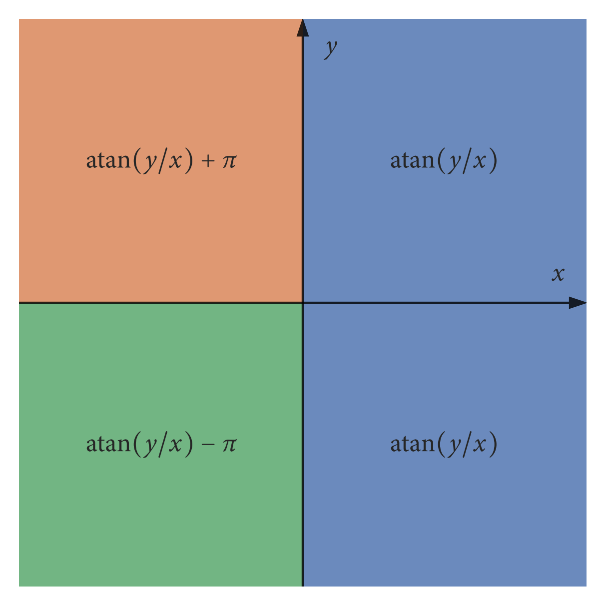
As an $AR(p)$ model:

$$\theta_{i,t+1} = \operatorname{atan2}\left(\sum_{k=1}^{p}\sum_{j=1}^{N}\varphi_{j,t-k+1}\omega_{ij,t-k+1}\sin\theta_{j,t-k+1},\right.$$
$$\left.\sum_{k=1}^{p}\sum_{j=1}^{N}\varphi_{j,t-k+1}\omega_{ij,t-k+1}\cos\theta_{j,t-k+1}\right) + \epsilon_{i,t}$$

As a topological model:

$$\theta_{i,t+1} = \operatorname{atan2}\left(\sum_{j\in\mathcal{N}_{i,t}}\sin(\theta_{j,t}) + n_i \bmod \lfloor n_i \rfloor \sin(\theta_{j_*,t}),\right.$$
$$\left.\sum_{j\in\mathcal{N}_{i,t}}\cos(\theta_{j,t}) + n_i \bmod \lfloor n_i \rfloor \cos(\theta_{j_*,t})\right) + \epsilon_{i,t}$$

Questions?



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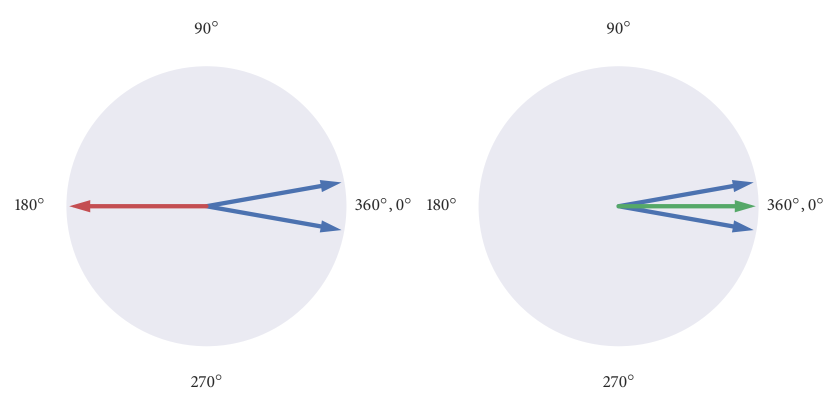
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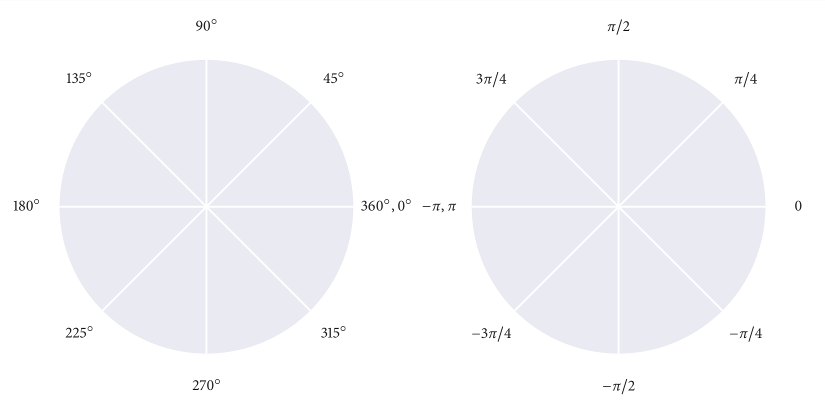
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